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On Modelling and Simulation of Electric Circuit Problems

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Abstract

Differential equations are of fundamental importance in Mathematics, Physical Sciences and Engineering Mathematics. Many mathematical relations and physical laws appeared in the form of such equations. This paper reviewed an application of these equations in solving mathematical model on electric circuit problems using the First order linear differential equation. The analytical approached in solving the equations confirmed that solving electric circuits using first order linear ordinary differential equations gives accurate and reliable result. Therefore, the application is of importance and great need. However, complex problems need higher order differential equations, which are nonlinear and have entirely different approach in finding their solutions.

Keywords: Differential Equation, First Order Linear Differential Equation, Circuit and Quantity of Current.

INTRODUCTION

An electric circuit is an interconnection of electric component such that electric charge is made to flow along a closed path (a current) usually to perform some useful task. A circuit is any closed path with an electric network, a circuit containing a source of electromotive force E (a battery or a generator), a resistor R, inductor L, a condenser (or capacitor) C and a switch all in series (Boyce et al., 2001; Dilwyn & Muke, 1989; Galadanci, 2014). Resistor is an electrical component that limits or regulates the flow of electrical current in an electronic circuit. The resulting current is inversely proportional to the resistance. This is ohm's law, which states that the current (I) is equal to the voltage (V) divided by the resistance (R) that is I=V/R. Inductor is a passive electronic component that stores energy in the form of a magnetic field. In its simplest form, an inductor consists of a wire loop or coil. Inductance is directly proportional to the number of turns in the coil, that is L. Capacitor is a small piece of electrical hardware that can hold electrical energy within a circuit or field. Also, capacitors are used in power supplies, amplifiers, Signal processors, oscillators and logic gate. Electromagnetic Force a fundamental force in nature, the electromagnetic force acts between charge particles and is the combination of all electrical and magnetic forces. Example, the electromagnetic force holds atoms together in molecules, causes friction and attracts iron to a magnet.

Mathematical modeling is a process by which a framework is constructed mathematically to represent real life situation. The frame work is further analyze with mathematical methods and techniques. Then translate and interpret the result back into the context of real life situation. The industries and trade have been modeled and computerized in modern time.

A model is defined as a simplified representation of certain aspect of life system. This may be physical, biological or economical. A model specifies problems, reduces complication and lead to easy solution of problems. This may be (i) Deterministic models or (ii) Stochastic models, (Dilwyn & Muke, 1989; Galadanci, 2014).

A differential equation is an equation containing one or more derivatives of the unknown functions with respect to one or several independent variables. The ***order*** of a differential equation is the order of the highest ordered derivative or differential coefficient that appears in the equation while the ***degree*** of a differential equation, is the highest exponent of the powers of the highest order derivatives appearing, (occurring) in the equation (Magaji et al., 2019; King et al., 2003; Al-Ahmad et al., 2019).

Studies show that in industries as well as commerce, the availability of fast and powerful computers have made it possible for mathematicians to solve a great range of problems previously difficult to solve because of their complexity. Engineers often use a mental model or theory to explain their observations, make predictions and then test their prediction by experiment. Mathematicians unlike other engineers do not deal directly with any physical objectives; their objects are ideas in mind, which have no physical existence. However, most of the mathematical ideas are formed in the mind through idealization of physical objects. The step leading from the physical object to mathematical ideas is called modeling (Galadanci, 2014).

This paper studied the application of first order linear ordinary differential equations in Engineering particularly in solving electrical circuits problems on which determines the quantity of current that pass through the circuit. The work is to review and apply some existing methods for Solving Mathematical Models involved in engineering problems particularly Electric circuit. The problems that will be considered and the models/equations arising from the problems are not newly formulated or modeled from a physical/real life situation or rather sourced from the workshop as a result of an experiment, but alternative way to solve such problems.

THE METHOD OF FINDING SOLUTION OF FIRST ORDER EQUATIONS

In this section, the methods for finding the solution of first order linear ordinary differential equations are presented. In each method, an example is given to demonstrate how to obtain the solution to such equations.

A. Method of Separable Variable

A first order differential equation is said to be variable separable if it can be written in the form

$$\frac{dy}{dx} = \frac{g(x)}{f(y)} \quad (1)$$

$$f(y)dy = g(x)dx \quad (2)$$

Then, integrate both sides of (2) to obtain the solution of the differential equation (1) as

$$F(y) = G(x) + C$$

Examples: Find the solution to the following differential equations

(a) $\frac{dy}{dx} = \frac{y-1}{x+1}$

(b) $(1 + x^2)\frac{dy}{dx} = 2xy$

Solutions

Given that $\frac{dy}{dx} = \frac{y-1}{x+1}$

Then separable the variable and integrate both side as

$$\int \frac{dy}{y-1} = \int \frac{dx}{x+1}$$

$$\ln(y-1) = \ln(x+1) + c$$

$$\ln(y-1) - \ln(x+1) = c$$

By law of logarithm, we have

$$\ln\left(\frac{y-1}{x+1}\right) = c$$
$$\frac{y-1}{x+1} = e^c \rightarrow \frac{y-1}{x+1} = k$$

Where $k = e^c$
For the solution of (b) $(1+x^2)\frac{dy}{dx} = 2xy$ and more on separable variable see Galadanci (2003) for details

B. Method of Integrating Factor
An equation of the form

$$\frac{dy}{dx} + P(x)y = Q(x) \qquad (3)$$

Is a linear differential equation of first order, where P and Q are functions of x. Multiply both sides of equation (3) by the integrating factor $\varphi = e^{\int p(x)dx}$to get:

$$\left(\frac{dy}{dx} + P(x)y\right)e^{\int p(x)dx} = \mathrm{Q}(x)e^{\int p(x)dx} + C$$

Example: Solve the equation$x\frac{dy}{dx} + 2y = x^3$
Solution
Given that $x\frac{dy}{dx} + 2y = x^3$
Dividing through by x to obtain the standard form as in (3) gives

$$\frac{dy}{dx} + \frac{2y}{x} = x^2 \qquad (4)$$

where$P(x) = \frac{2}{x},\ Q(x) = x^2$
So, the integrating factor is given by

$$e^{\int p(x)dx} = e^{\int \frac{2}{x}dx} = e^{2lnx} = e^{lnx^2} = x^2$$

Now, multiplying both sides of equation (4) by the integrating factor, we obtain

$$\frac{d(yx^2)}{dx} = x^4 \qquad (5)$$

Integrating both sides

$$yx^2 = \frac{x^5}{5} + C \qquad (6)$$

Multiply (6) through byx^{-2}

$$y = \frac{x^3}{5} + Cx^{-2}$$

APPLICATION OF FIRST ORDER LINEAR ORDINARY DIFFERENTIAL EQUATION ON ELECTRIC CIRCUIT

Problem 3.1: In a model of electric circuit below. The figure shows a circuit containing an electromotive force, a capacitor with a capacitance of C farads (F) and a resistor of R ohms (Ω). A review shows that the voltage drop across the capacitor is $\frac{Q}{C}$, where Q is the charge (in coulombs), so in this case Kirchhoff's law gives:

$$RI + \frac{Q}{C} = E(t) \qquad (7)$$

But,$I = \frac{dQ}{dt}$ so we have

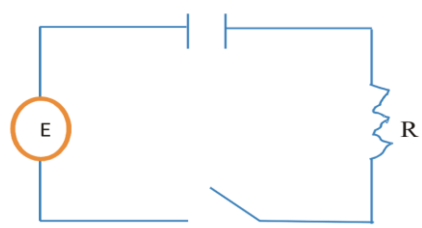

Fig. 3.0: A circuit containing an electromotive force, a capacitor with a capacitance of C farads (F) and a resistor of R ohms (Ω).
Suppose the resistance is 5Ω, the capacitance $0.05F$, a battery gives a constant voltage of $60V$ and the initial charge is $Q(0) = 0$.
(a) Find the charge and current at time t.
(b) What is the charge at $t = 3s$?

Solution
Give that $RI + \frac{Q}{C} = E(t)$ where $R = 5, C = 0.05, E(t) = 60$
Therefore,
$$5\frac{dQ}{dt} + \frac{1}{0.05}\mathrm{Q} = 60 \quad (8)$$
Dividing both sides by 5
$$\frac{dQ}{dt} + 4\mathrm{Q} = 12 \quad (9)$$
Applying the integrating factor
$$\varphi = e^{\int p(t)dt} = e^{4\int dt} = e^{4t}$$
Multiply the integrating factor to both side of equation 9
$$e^{4t}\frac{dQ}{dt} + e^{4t}\frac{\mathrm{Q}}{4} = 12e^{4t} \quad (10)$$
$$\frac{d}{dt}(e^{4t}Q) = 12e^{4t}$$
Integrating both sides,
$$e^{4t}Q = 12\int e^{4t}\,dt$$
$$e^{4t}Q = 12\left(\frac{e^{4t}}{4}\right) + C$$
$$e^{4t}Q = 3e^{4t} + C \quad (11)$$
Multiply (11) through bye^{-4t}
$$Q = 3 + Ce^{-4t} \quad (12)$$
$$Q(0) = 3 + C$$
Therefore,$C = -3$
$$Q(t) = 3 - 3e^{-4t}$$
$$Q(3) = 3 - 3e^{-4(3)}$$
$$= 3 - 3e^{-12} = 2.99 \approx 3columbs$$

Problem 3.2:
A generator supplies a voltage of $E(t) = 40sin60$ volts, the inductance is 1H, the resistance is 20Ω and $I(0) = 1$. Find;
(a) Find *I(t).*
(b) Find the current after 0.1s.

Solution
The differential equation for the system is given by:
$$L\frac{dI}{dt} + RI = E(t) \quad (13)$$
From the given problem, $L = 1, R = 20, I = 1, E(t) = 40sin60t$.
$$\therefore \frac{dI}{dt} + 20I = 40sin60t \quad (14)$$
The integrating factor is:
$$\mu = e^{\int Pdt} = e^{\int 20dt} = e^{20t}$$
Multiplying equation (14) through by the integrating factor,
$$e^{20t}\frac{dI}{dt} + 20e^{20t}I = e^{20t}40sin60t$$
$$\rightarrow \frac{d}{dt}(e^{20t}I) = 40e^{20t}sin60t \quad (15)$$
Integrating both sides of the equation 3.9
$$e^{20t}I = 40\int e^{20t}sin60tdt \quad (16)$$

Let $y = \int e^{20t} sin60tdt$ (17)

By applying the integration by parts, $U = e^{20t}$, $dV = sin60tdt$, $V = -\frac{cos60t}{60}$, $dU = 20e^{20t}$

So, $\int UdV = UV - \int VdU$ (18)

$= -\frac{e^{20t}cos60t}{60} + \frac{20}{60}\int e^{20t}cos60tdt$ (19)

$y = -\frac{e^{20t}cos60t}{60} + \frac{1}{3}\int e^{20t}cos60tdt$

We again integrate $\int e^{20t}cos60tdt$ by parts to have

$\int e^{20t}cos60tdt$

Let U = e^{20t}, dV = cos60tdt, V = - $\frac{sin60t}{60}$, dU = 20e^{20t}

So, $\int e^{20t}cos60tdt = \frac{e^{20t}sin60t}{60} - \frac{20}{60}\int e^{20t}sin60tdt$ (20)

But y = $\int e^{20t}sin60tdt$, therefore (20) becomes

$= \frac{e^{20t}sin60t}{60} - \frac{1}{3}$ y (21)

Therefore, $y = -\frac{e^{20t}cos60t}{60} + \frac{1}{3}\left[\frac{e^{20t}sin60t}{60} - \frac{1}{3}y\right]$

$y = -\frac{e^{20t}cos60t}{60} + \frac{e^{20t}sin60t}{180} - \frac{1}{9}y$

$y + \frac{1}{9}y = -\frac{e^{20t}cos60t}{60} + \frac{e^{20t}sin60t}{180}$

$\frac{10}{9}y = -\frac{e^{20t}cos60t}{60} + \frac{e^{20t}sin60t}{180}$ (22)

Multiply equation 22 through by 9 to obtain

$10y = -\frac{9e^{20t}cos60t}{60} + \frac{9e^{20t}sin60t}{180}$

$10y = \frac{e^{20t}sin60t}{20} - \frac{e^{20t}cos60t}{60}$ (23)

Dividing both sides of (23) by 10

$y = \frac{e^{20t}sin60t}{200} - \frac{e^{20t}cos60t}{600}$

$y = \frac{3e^{20t}sin60t - 9e^{20t}cos60t}{600}$ (24)

By substituting equation 24 into equation 26, therefore equation (26) becomes

$e^{20t}I = 40e^{20t}\left[\frac{3e^{20t}sin60t - 9e^{20t}cos60t}{600}\right] + C$

$e^{20t}I = e^{20t}\left[\frac{3e^{20t}sin60t - 9e^{20t}cos60t}{15}\right] + C$ (25)

Multiply equation (25) through by e^{-20t}, we get

$I = \frac{3e^{20t}sin60t - 9e^{20t}cos60t}{15} + Ce^{-20t}$ (26)

Now at t = 0

$I(0) = \frac{-9}{15} + C = 0, \quad \therefore C = \frac{9}{15}$

(a) $I(t) = \frac{3e^{20t}sin60t - 9e^{20t}cos60t}{15} + Ce^{-20t}$

(b) $The\ Current\ after\ 0.1s$

$I(0.1) = \frac{3sin60(0.1) - 9cos60(0.1)}{15} + \frac{9}{15}e^{-20(0.1)}$

$I(0.1) = \frac{3sin6 - 9cos6}{15} + \frac{9}{15}e^{-2}$

$I(0.1) = -0.5758 + 0.0812$

$= 0.4946\ Columbs$

RESULT AND DISCUSSION

The first order differential equations, which are the simplest forms of ordinary differential equations, plays an important role in different fields of Engineering when formulated from a real or physical situation. Differential equations are of fundamental importance in Mathematics, Physical Sciences and Engineering

Mathematics. Many mathematical relations and physical laws appeared in the form of such equations. In this paper, the application is studied, problems related to electrical circuits are presented, and solution to these problems is provided.

CONCLUSION AND RECOMMENDATION

Based on the problems and method discussed, it is recommended that applying first order linear differential equations in solving problems related to electric circuits is suitable and easier than the direct method. The analytic approach used in solving the equations confirmed that solving electric circuits using first order linear ordinary differential equations gives accurate, efficient and reliable result. However, complex problems need higher order differential equations, which are nonlinear and have entirely different approach in finding their solutions. Therefore, the applications is of importance and great need.

References

Al-Ahmad S, Sulaiman I M, Al-Ahmad R, Mamat M and Al-Ahmad M. (2019). On Solving Classes of Differential Equations with Applications, Malaysian Journal of Computing and Applied Mathematics, 2(2), 8-13.

Boyce W E and DiPrima R C. (2001). *Elementary Differential Equations and Boundary Value Problems*, Seventh Edition, John Wiley & Sons, Inc; United States of America, Chapter 2, pp 45-60.

Dilwyn E and Muke H. (1989). *Guide to Mathematical Modeling*, Macmillan Press Ltd.

Galadanci G S M. (2003). *Introduction to Electricity and Magnetism for Scientist and Engineers,* A.B.U Press limited, Zaria, Kaduna State Nigeria.

Galadanci G S M. (2014). *Introduction to Electricity and Magnetism for Scientist and Engineers (Revised edition),* A.B.U Press limited, Zaria, Kaduna State Nigeria.

King A C, John B. and Otto S R. (2003)*. Differential Equations; Linear, Nonlinear, Ordinary and Partial:* Cambridge University Press.

Magaji A S, Nasir M and Dauda M K. (2019). Mathematical Modeling of Hepatitis B In Kaduna Metropolis Using Euler Method, *Journal of Science, Technology, Mathematics and Education (JOSTMED),* 15(1), 99-118.